STATISTICS FOR ROOT NUMBERS OF ELLIPTIC CURVES

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Let E be an elliptic curve defined over $\mathbb{Q}$. For each prime number p (including the prime at infinity), one associates to E the local root number $W_p(E)$.This is equal to $+1$ at primes of good or non-split multiplicative reduction, -1 at primes of split multiplicative reduction, and at primes of additive reduction is ± 1. Rohrlich, [4], explained how to find $W_p(E)$ for every $p \neq 2, 3$. Explicit formulas were given in [3] for characteristics 2 and 3. You may also want to see [1].

The root number of E is defined as $W(E) = \prod_{p \leq \infty} W_p(E)$. In view of the Birch-Swinnerton-Dyer conjecture, the parity of the Mordell-Weil rank, $r(E)$, of E is governed by $W(E)$. In other words, the Parity Conjecture states that $(-1)^{r(E)} = W(E)$.

In this project, fixing a prime number p, we aim to compute the density of elliptic curves with a given local root number at p. This will require careful analysis of the local conditions forcing the local root number to be either -1 or $+1$. This will be followed by implementing counting techniques like the ones in [2].

Another question that we will try tackling during the project is if it is feasible to construct a family of elliptic curves with a fixed root number and of positive density. Obtaining such global densities from local information requires the use of Ekedahl Sieve.

Participants may refresh their background on the arithmetic of elliptic curves by reading through [5, 6].

References

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